Physics Graduate School Qualifying Examination

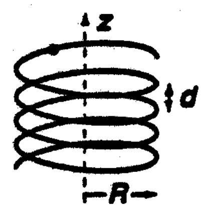
January 7, 1999

Part I

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your 3-digit student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

I-1. A ball is placed at the top of an inclined plane which makes an angle θ relative to the horizontal. If the ball is released from rest, what is the minimum value of coefficient of static friction between ball and plane such that the ball will roll down the plane without slipping?

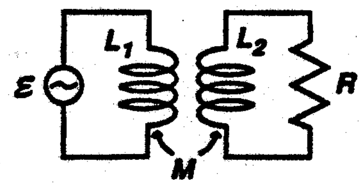
I-2. A bead with mass m slides under the action of gravity down a frictionless wire made into a right circular spiral with radius R and step d. The axis of the spiral is vertical, and is chosen as the z-axis (with the positive direction chosen upward).



a Determine the equation of motion for the z coordinate of the bead.

b Determine the time it takes for the bead to move down a distance h, assuming that it starts from rest.

I-3 An ideal transformer consists of two inductors of self inductance L_1 and L_2 with mutual inductance $M = \sqrt{L_1 L_2}$. Their ohmic resistance is negligible. A resistance R is connected to the terminals of L_2. What is the impedance of the system if a power source of angular frequency ω is connected to the terminals of L_1.



I-4. A sherical shell with a uniform fixed surface charge density σ rotates about a diameter with angular speed ω. Calculate the magnetic field B at the center of the shell.

I-5. Three coordinate frames, A, B and C, with parallel axes, are in uniform relative motion with respect to each other. B moves relative to A with speed v_0 in the $+x$-direction. C moves relative to B with speed v_0 in the $+y$-direction. An object with rest mass m_0 moves relative to C with speed v_0 in the $+z$-direction. What is the velocity of the mass relative to A?

I-6. The Bohr-Sommerfeld quantization rule states that the action integral over a complete cycle of a system's motion is quantized,

$$\oint p(x)dx = nh$$

Use this result to calculate the allowed energy levels of a ball which is bouncing elastically in a vertical direction.

I-7. Two simple pendulums oscillate with small amplitudes in vacuum; each consists of a charged mass at the end of a string. The charged masses are identical. The fraction of the total energy lost per period into radiation is measured for each pendulum. If one pendulum has a string which is twice as long as the other, what is the ratio of these fractions?

I-8. A point particle with mass m and charge q drops from rest under the influence of a uniform gravitational field which points in the negative z-direction. A uniform, horizontal B-field (in the $+x$-direction) is also present. Assume that the particle starts at the origin at $t=0$.

a Derive an expression which gives its position as a function of time.

b Give a qualitative description of the subsequent motion.

Physics Graduate School Qualifying Examination

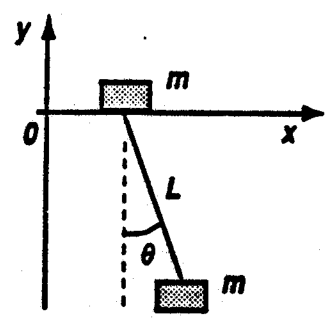
January 8, 1999

Part II

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your 3-digit student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

II-1. A bucket of mass m is pulled up a well by a rope which exerts a constant force F on the bucket. Initially, the bucket is at rest and contains a mass of water M_0. The water leaks out at a constant rate at zero velocity relative to the bucket. It becomes empty at time T before it reaches the top of the well. Calculate the speed of the bucket at time T.

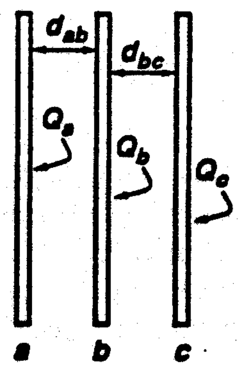
II-2. Two identical small bodies of mass m are connected by a massless flexible string of fixed length L. They are constrained to move in the vertical (x,y) plane; one of the masses is further constrained to move, without friction, along the x-axis. The other mass moves as a part of a simple pendulum.



a Determine the equations of motion in terms of the coordinates, x (for the top mass) and θ.

b Determine the frequency of small oscillations for the system.

II-3. Three large, identical, parallel, conducting plates a, b and c, each with area A, carry charges Q_a, Q_b and Q_c, respectively. The distances between the plates are d_{ab} and d_{bc}. For this distribution of charges, the potential of plate a relative to plate b is observed to be V_{ab}.



a. Find the electric charges on each of the six surfaces.

b. Find the potential of plate b relative to plate c, V_{bc}.

II-4. An electromagnetic wave with harmonic time dependence of frequency $\omega / 2\pi$ propagates in a medium of constant electric and magnetic permittivities, ε and μ, and conductivity σ.

a. Show that as $t \to \infty$, the charge density $\rho \to 0$.

b. For a poor conductor, with $\sigma \ll \varepsilon\,\omega$, determine the spatial distance over which the wave amplitude is attenuated by a factor of 1/e.

II-5. An electron in a hydrogen atom is in a 3d state.

a. If it is in a state with a definite value for the magnetic quantum number, m_l, what are the possible values for the angle between the orbital angular momentum vector and the quantization direction?

b. What are the possible values for the magnitude of the total angular momentum associated with this electron?

c. For each of your answers in part b, what are the possible values for the m_j quantum number?

II-6.

a Show that the conversion of a photon into an electron–positron pair ($\gamma \rightarrow e^+ e^-$) cannot occur in vacuum.

b If a photon passes near a nucleus N, the conversion process $\gamma N \rightarrow e^+ e^- N$ can occur, provided that the energy of the photon exceeds $2m_e c^2$ by a certain minimal amount. Find that minimal amount, assuming that the nucleus is initially at rest, and its mass M is much larger than the electron mass m_e.

II-7. Within the framework of the Bohr theory of the hydrogen atom, calculate the magnetic induction, B, at the position of the nucleus of the hydrogen atom. Assume that the hydrogen atom is in its ground state. Ignore electron spin.

II-8. The sun has radius R and surface temperature T. According to Stefan's law, a black body emits radiant energy (photons) at a rate given by $P = A\sigma T^4$. A thin, flat solar sail of area ΔA and mass M is situated at a distance d from the sun. The sail is a perfect mirror, reflecting all photons incident upon it.

a If the sail is oriented to maximize the number of photons hitting it, what is its acceleration? (Give magnitude and direction!)

b Assume the sail is reoriented so that it has the maximum possible component of acceleration in a direction perpendicular to a line drawn from the sail to the sun. What is the value of this maximum acceleration component?