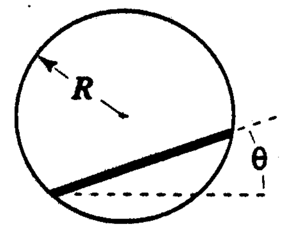
Physics Graduate School Qualifying Examination

August 20, 1998

Part I

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your 3-digit student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

I-1. A uniform, thin, straight bar of length L and mass m is constrained to move without friction in a fixed vertical circular frame of radius R.

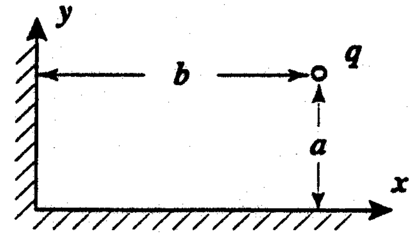


a Determine the kinetic and potential energies of the bar in terms of m, L, R, the gravitational acceleration g, and θ, the angle between the bar and the horizontal.

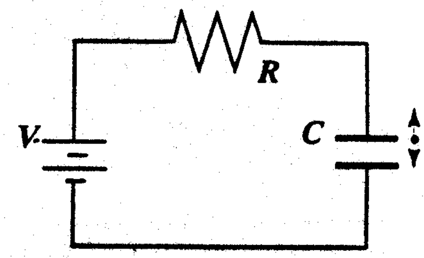
b Determine the frequency for small oscillations of the bar about its equilibrium position.

I-2. A river of width L flows southward with a speed v at a latitude θ above the equator. What is the difference in height of the water at the two river banks? At which bank (east or west) is the water higher?

I-3. A point charge q lies near the intersection of two perfectly conducting, grounded, perpendicular, semi-infinite planes. The perpendicular distances to the planes are a and b as shown. Calculate the external work required to move the charge to a position far from the conducting planes.



I-4. An air capacitor of area A and equilibrium separation d_0 is connected in series with a resistor R. A constant EMF V is applied in series with both. One plate of the capacitor is mechanically driven so that the separation d between the plates varies sinusoidally



$$d = d_0 + \delta_0 \cos \omega t \quad \text{where} \quad \delta_0 << d_0.$$

After transients die out, what is the charge on the capacitor as a function of time to first order in δ? Assume $Q = Q_0 + q(t)$.

I-5. A spaceship has a searchlight that shines a cone of light in the forward direction. The cone has an opening half-angle α in the rest frame of the ship. What opening half-angle will be measured by an observer who sees the ship moving at a relativistic speed v?

I-6. A spinless particle with mass m moves in a three-dimensional cubical box of edge length a.

a What are the possible values for the energy of this particle?

b Estimate the number of different states which are possible with $E \leq E_0$ when

$$E_0 >> \frac{h^2}{ma^2}.$$

I-7. A particle with charge q and mass m travels through a uniform magnetic field of magnitude B in the $+z$-direction. At $t = 0$, the particle passes through the origin with speed v_0 in the $+x$-direction. Assume that a linear resistive force $\mathbf{F_R} = -\gamma\mathbf{v}$ is present, and that gravitational forces may be neglected. Show that the particle ultimately comes to rest, and find its final position.

I-8. A thin copper ring of radius a and mass m rotates about a diameter perpendicular to a uniform magnetic field $\mathbf{B}$. The initial angular velocity is ω_0. Assume that mechanical energy is lost only to Joule heat, and that this loss results in a slow change of angular velocity with time.

a Calculate the amount of mechanical energy lost to Joule heat during one period of rotation, at a time when the angular velocity is ω. The mass density of copper is ρ, and the electrical conductivity is σ.

b Find the angular velocity as a function of time. (The moment of inertia of a ring about its diameter is $^1/_2\, ma^2$.)

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Part II

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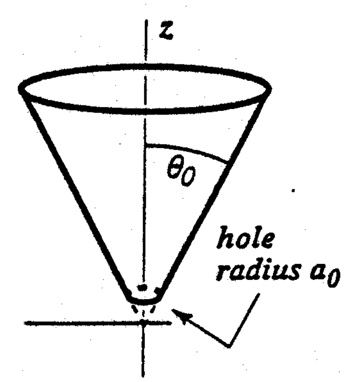
II-1. A wheel (mass M, radius R, moment of inertia about its axis I_0) is suspended at its rim by four equally spaced vertical ropes of length l. The wheel hangs in a horizontal plane. The wheel is rotated through a small angle about its axis and released.

a What is the period of small oscillations for the ensuing motion?

b If the wheel is constructed of three elements – a thin ring and two thin perpendicular crossbars which intersect at the wheel's center – each of mass m, what are I_0 and ω?

II-2. A point mass, m, moves on the surface of a cone whose symmetry axis is vertical (along the z-axis) and has a half-opening angle θ_0. At the bottom of the cone, a horizontal cut takes off the tip leaving a small hole of radius a_0. Motion of the mass takes place under the influence of gravity.



a Write down the Lagrangian for the mass m.

b Find the equations of motion, and determine the constants of the motion.

c Assume that the mass has an initial energy E_0. (The mass would have zero energy if it was at rest at the vertex of the cone.) Find the condition for which the mass m, starting above the hole, will not fall into it.

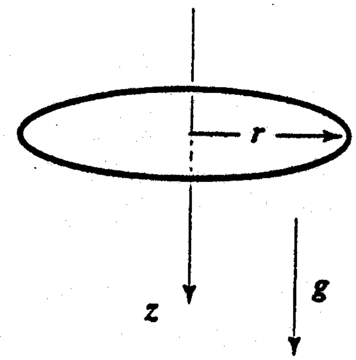
II-3. A superconducting circular loop of radius R is made from wire with cross-sectional area A. The loop is released into a magnetic field specified by

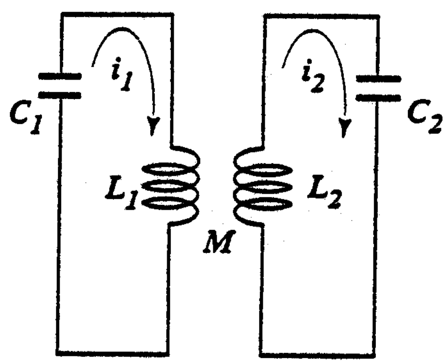
$$\mathbf{B}(r,z) = \beta(z\hat{\mathbf{z}} - \tfrac{1}{2}r\hat{\mathbf{r}})$$

where the z-axis is directed downward (as is gravity), and β is a positive constant. Assume that the loop lies in a horizontal plane, and is initially at rest with its center at the origin and carries zero current. Use the London Equation,

$$\frac{\partial \mathbf{J}}{\partial t} = a\mathbf{E}$$

and Maxwell's equations to find the equilibrium position of the wire loop. (In the above equation, $\mathbf{J}$ is the current density in the wire, $\mathbf{E}$ is the electric field and a is a positive constant.)





II-4. Two ideal L-C circuits are coupled by their mutual inductance, M, as shown in the diagram.

a Determine the differential equations for the currents i_1 and i_2.

b Calculate the frequencies of the normal mode oscillations of this system.

II-5. Special relativity predicts transverse and parallel Doppler shifts when we observe electromagnetic radiation emitted by a moving source. Use the Lorentz transformation to derive these Doppler shifted wavelengths for light emitted isotropically from a source which moves with speed v

a towards the observer (parallel Doppler shift),

b perpendicular to a line drawn from source to observer (transverse Doppler shift).

Assume that λ_0 is the wavelength of the light in the source's rest frame. Express your answer in terms of $\beta = {}^{v}/_{c}$.

II-6. N non-interacting identical particles of mass m are confined to a one-dimensional box extending from $x = -L/2$ to $x = +L/2$. Assume that N is an even integer.

a If the particles have zero spin, determine the ground and first excited state energies.

b If the particles have spin one-half, determine the ground and first excited state energies.

c Determine the magnitudes of the forces on the box walls for parts a and b.

II-7. In the Millikan experiment, a charged oil drop was suspended at rest between two capacitor plates which are separated by a distance d and maintained at a potential difference V_0. When the potential difference was removed, the drop's terminal velocity in air was measured to be v_T. Assume that the viscous drag force on a sphere moving through air is given by $F = 6\pi\eta r v$ where η is the coefficient of viscosity, and r and v are the radius and speed of the sphere. Obtain an expression for the electric charge on the drop in terms of the given quantities, the acceleration of gravity, and the densities of oil and air.

II-8. A small particle of mass m and electric charge q is attached at $x = 0$ to a long string with tension T and mass per unit length σ. The string is initially at rest along the x-axis. At $t = 0$, an oscillatory electric field $E_0 \cos \omega t$ is applied in the y-direction. Neglect gravitational effects and reflections at the ends of the string.

a Determine the partial differential equation which describes small transverse vibrations in the y-direction for $x \neq 0$.

b Determine the general functional form of the transverse string displacement $y(x, t)$ for $x > 0$ and $x < 0$.

c Determine the differential equation for the particle displacement $y(0, t)$ for $t > 0$, and solve the equation for the case of $m = 0$.