Recap

• In 1820, Hans Oersted observed that a current carrying wire (moving charges) produced a magnetic field.

• If a moving charge experiences a force in a magnetic field, will a moving magnetic field exert a force on a stationary charge?

• Yes... but we don’t quite describe it this way.

• A changing magnetic field creates an electric field which changes the electrical potential energy of charge carriers in a circuit.

• A changing magnetic field induces an electromotive force in a circuit.
Magnetic Induction

• In 1831, Faraday discovered that a changing magnetic field creates an electric field.

  – This effect is called *magnetic induction*
  – Faraday’s discovery couples electricity and magnetism in a fundamental way

• Magnetic induction is the key to MANY technologically relevant inventions.
Magnetic Flux

• Another term for the “magnetic field” is **magnetic flux density**

• Magnetic flux is the “number” of magnetic field lines intersecting a surface:

\[
\Phi_B = B \cdot A
\]

Uniform magnetic field \(\vec{B}\), surface area \(A\) produces magnetic flux \(\Phi_B = B \cdot A\)

MKS units for magnetic flux is the Weber (Wb)
Magnetic Flux

- Magnetic flux is the product of the area with the component of the magnetic field passing perpendicularly through it.

\[ \Phi_B = B \cdot A \cdot \cos \theta \]

\( \theta \) measures the angle between \( \vec{B} \) and \( \vec{A} \).

Units are \( T \cdot m^2 = 1 \text{ Weber} = 1 \text{ Wb} \)
Why $\cos \theta$?

- $\vec{B}$ is a vector!
- $\vec{A}$ is $\perp$ to green plane

$\vec{B}_{\perp} = B \cos \theta$
Magnetic Flux

- The orientation of the surface is important!
- Surfaces have two sides...
  - Draw a loop around the boundary
  - Use the right-hand rule to see what direction your thumb is pointing
  - If your thumb points in the same direction as $\vec{B}$ then the flux is positive
  - If your thumb points in the direction opposite $\vec{B}$ then the flux is negative
Magnetic Flux

\[ \Phi_B > 0 \]

\[ \Phi_B < 0 \]
Examples of changing magnetic flux

\[ \Delta \Phi_B < 0 \]

\[ \Delta \Phi_B > 0 \]
Faraday’s Law – Electromagnetic Induction

• Faraday described many magnetic effects on circuits in terms of magnetic flux:

\[ \Phi_B = \sum \vec{B} \cdot \Delta\vec{A} \]

• Faraday’s law: \( \mathcal{E}_{ab} = -\frac{\Delta\Phi_B}{\Delta t} \)

• The minus sign is determined using *Lenz’s Law*...
Faraday's Law: An induced emf is produced in a coil whenever the magnetic flux changes with time. Here we focus on an increasing flux through a loop.

1. $B$ is an external applied magnetic field that changes with time. You control $B$!

2. $\mathcal{E}$ is the induced emf that develops when the magnetic flux thru the loop increases with time. This $\mathcal{E}$ drives the current $I$.

3. $I$ is the induced current that flows due to $\mathcal{E}$

Key Idea: An emf is produced by the changing magnetic flux. The emf in turn produces an induced current $I$.

Subtle Point: The negative sign indicates that the induced emf is opposite to change in magnetic flux that causes it.
Review: You need to know the direction of the magnetic field produced by current in a wire.
Lenz’s Law

• Magnetic fields are like mass in mechanics – they have inertia and would prefer to remain constant.

• Any a changing magnetic field induces an electromotive force.

• The electromotive force would cause current to flow in the direction that tries to keep the magnetic field constant.
IMPORTANT

\[ E = -\frac{\Delta \Phi_B}{\Delta t} \]

Minus sign is due to Lenz’s Law

Four ways the flux can change:

1. Change $B$

2. Change area

3. Rotate loop

4. Move the loop

- B constant but area $A$ changes with time: $A = \ell w$
- where $\ell = v \Delta t$
- Loop moves from one region to another and magnitude of $B$ field is different
Example: What is the induced emf if the magnetic field through a **six** turn coil **increases** at a rate of 0.17 T/s?

\[ \varepsilon = - \frac{\Delta \Phi_B}{\Delta t} \]

**The negative sign indicates that the induced emf acts to "oppose" the change in magnetic flux that causes it.**

\[ |\varepsilon| = \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta (\vec{B} \cdot \vec{A})}{\Delta t} = A \frac{\Delta B}{\Delta t} \]

\[ A = \pi R^2 = 0.88 \text{ m}^2 \]

\[ \frac{\Delta B}{\Delta t} = +0.17 \text{ T/s} \]

\[ |\varepsilon| = (0.88 \text{ m}^2)(0.17 \text{ T/s}) = 0.15 \text{ V} \]

Since coil has six turns, \( E = 6 \times (0.15 \text{ V}) = 0.90 \text{ V} \)

It is often easier to take the absolute value of Faraday’s Law to find the **magnitude** of the induced emf and then use Lenz’s Law to find the **direction** of the induced current that results.
A. Assume a metal loop in which the applied magnetic field (solid arrows) passes upward through it
B. Assume the magnetic flux increases with time
C. The **induced** magnetic field produced by the **induced** current must oppose the change in flux
D. Therefore, the induced magnetic field (dotted arrows) must be downward
E. The induced current must therefore be clockwise (CW) when viewed from the top of coil
The direction of the magnetic field is always down.

The magnitude increases, then decreases.

The situation when the North pole \textbf{just} enters the loop

Be able to distinguish the applied field from the induced field

An induced current must flow in the loop to produce a magnetic field (inside the loop) that opposes the change in flux.
Lenz’s Law - Example 3 - Motion to and from a loop

(Focus on the Change in Flux w. Time)

a) Steady motion toward loop

\[ \Phi_B \]

\[ \text{increasing time} \]

\[ I_{\text{ind}} \]

b) Steady motion away from loop

\[ \Phi_B \]

\[ \text{increasing time} \]

The induced current always produces a magnetic field that opposes (counteracts) the change in flux.
Lenz’s Law

There are four cases to consider for one loop!

If $B_{\text{applied}}$ increases with time

- $I_{\text{induced}}$ is $\text{CCW}$

If $B_{\text{applied}}$ decreases with time

- $I_{\text{induced}}$ is $\text{CW}$

If $B_{\text{applied}}$ increases with time

- $I_{\text{induced}}$ is $\text{CW}$

If $B_{\text{applied}}$ decreases with time

- $I_{\text{induced}}$ is $\text{CCW}$